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Thermostimulated domain wall quantum tunnelling in the high-temperature region

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Abstract. A theoretical and numerical investigation of the quantum tunnelling of the domain walls in ferromagnets and weak ferromagnets has been performed, taking into account the interaction between the walls and the thermal excitations of the crystal. A numerical method for making calculations of the probability of thermally stimulated quantum depinning as a function of temperature has been evolved.

1. Introduction

Macroscopic quantum effects in magnetism are currently of great interest. Such phenomena are important in tests of quantum mechanics [1]. In particular, magnetic domain wall tunnelling seems to be one of the most appropriate subjects for investigation in this field. Together with detailed theoretical and experimental research into domain walls in ferromagnets [2–5, 14], recently similar phenomena occurring in weak ferromagnets were described [6].

For the description of the domain wall dynamics, it is convenient to apply a model in which the wall is considered as a quasiparticle with a certain effective mass m. Such a quasiparticle transfers, via the crystal, a change in magnetic moment orientation. In the movement through the crystal, the quasiparticle can be trapped by a magnetic pinning centre—as produced, for example, by an impurity raising the anisotropy energy locally. The domain wall can then overcome this energy barrier in the following ways: by absorption of an external field's energy, or due to thermal activation, or via tunnelling.

Tunnelling and thermal activation are usually considered as competing processes; as a result, one might be led to think that tunnelling would be observable only at extremely low temperatures, of about 0.001–1 K, whereas at higher temperatures tunnelling would be suppressed by thermal activation. However, this is not always the case. The purpose of this paper is to carry out a theoretical and numerical investigation of the situations in which to some extent the two above-discussed phenomena can cooperate. One way of thinking of this is to assume that, due to interaction with thermal excitations of the crystal and absorption of their energy, the wall will be 'raised' in front of the barrier. As a result, the effective height of the barrier will decrease and, accordingly, the tunnelling rate will increase. Also, we give a detailed discussion of this mechanism for the Bloch walls and walls in weak ferromagnets.

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2. Thermally activated tunnelling of the Bloch walls

2.1. The model and equations of motion

Let us consider first a 180° Bloch wall in a uniaxial crystal. The form of the wall at rest is given by the well-known Landau–Lifshitz [9] exact solution

$$\sin\theta = \tanh(x/\sqrt{A/K_1}) \tag{1}$$

where x is directed along the easy axis, A is the exchange constant and K_1 is the uniaxial anisotropy constant. For a travelling wall an additional 'kinetic' energy K arises. This may be represented in a form that is quadratic in the velocity v, namely $K = v^2 m_d/2$, where $m_d = E_g/8\pi A\gamma^2 \sin^2\theta$ is the so-called Döring mass [8], or the surface density of the mass, and $E_g = \sqrt{AK_1}$ is the surface energy density of the wall. On this basis, the equation of motion takes the Newton form:

$$m_d \frac{\mathrm{d}^2 X}{\mathrm{d}t^2} = 2I_s H$$

where X is the coordinate of the centre of the wall. Let us consider interaction between the wall and a defect, introducing the potential energy $U(X)$; then the latter equation takes the form

$$m_d \frac{\mathrm{d}^2 X}{\mathrm{d}t^2} = 2I_s H - \frac{\mathrm{d}U(X)}{\mathrm{d}X}. \tag{2}$$

In this paper we consider point-like repulsive impurity. In this case the potential energy of the wall will change according to $U(\theta) = U_0 \cos^2\theta$, where U_0 is the maximal value of the potential energy attainable when the impurity lies at the centre of the wall. If one takes into consideration equation (1), it is easy to obtain the explicit function $U(X)$:

$$U(X) = U_0(1 - \tanh^2(X/\sqrt{A/K_1})) = U_0 \cosh^{-2}(X/a) \tag{3}$$

where $a = \sqrt{A/K_1}$ is the characteristic width of both the domain wall and the potential $U(X)$.

In deriving equation (2), it was assumed that the structure of the moving wall does not vary and that the definition of m contains the energy of the rest wall. However, Walker [7] has shown that this assumption is justified only for slowly travelling walls. As follows from exact solutions of the equation of motion for the Bloch wall, the structure of a wall (in particular, its width and mass) strongly depends on the velocity of the movement. When the wall velocity tends to some critical value c, the derivative of the wall energy reduced to infinity, its mass tends to infinity and its width tends to zero. Usually, for normal ferromagnets, the Walker limiting velocity c has a value around several kilometres per second, and in this case $mc^2 \gg U_0$. For this reason, one may assume that the condition $v^2/c^2 \ll 1$ is usually satisfied for ferromagnets; therefore, in the current section, we shall limit consideration to the case of small velocities.

In the context of the problem under discussion, the following physical situation will be of interest to us. Let a domain wall with the kinetic energy K arrive at a potential barrier with the height U_0, which simulates its interaction with a defect of the crystal. If $K < U_0$, the segment of wall in the immediate vicinity of the barrier will be trapped in a metastable minimum. It is acceptable to consider such a segment as an isolated quasiparticle, with the effective mass determined as an integral over the area of the defect [4].

As stated above, there are three ways for the wall to overcome barrier: with the help of an external field, due to thermal activation and via tunnelling. Let us consider a very weak field

$$H \ll \frac{1}{I_s}\left|\frac{\partial U}{\partial X}\right|.$$

Such a field cannot disengage the wall, but it will create asymmetry for displacements of the wall in front of the barrier and behind the barrier. For the wall structure to remain unchanged, the condition $U_0 \ll mc^2$ must be satisfied. On the other hand, the height of the barrier should not be too small, or thermal activation will prevail and the tunnelling will be suppressed. Together, these restrictions lead to a rather narrow range of reasonable parameters. Nevertheless, the values which are acceptable are normal for ferromagnets; therefore it is possible to use them for the calculations. We will assume a height of barrier $U_0 = 10^{-14}$ erg, a quasiparticle mass $m = 10^{-26}$ g, a width of potential $a = 10^{-6}$ cm and a defect area $S = 10^{-13}$ cm^2.

2.2. Calculations and results

By virtue of the prior assumption of the weakness of the external field, it is appropriate to assume that the width of the metastable minimum is much greater than the width of the barrier, whereas its depth is considerably smaller than the barrier height. In such a case, the quasiparticles in front of the barrier have a quasi-continuous spectrum and using a Maxwellian distribution for the analysis of the problem is acceptable.

Let us consider a quasiparticle in thermal equilibrium with a crystal. We consider an ensemble containing N such particles. We shall perform computations according to the following computational scheme. The interval of energy from 0 up to $U_{\max}$† was divided into equal subintervals of width δw. The number of particles in each subinterval can be found from the expression

$$N_w = \int_{w-\delta w/2}^{w+\delta w/2} \frac{2N}{\sqrt{\pi}}(k_B T)^{-1.5}\sqrt{w}\exp\left(-\frac{w}{k_B T}\right)\,\mathrm{d}w. \tag{4}$$

In the next step we assign to all of the particles within the given subinterval the same average value w. The validity of this procedure was checked numerically, i.e. the number of divisions N was adjusted so as to guarantee the stability of the computation scheme as a whole.

Furthermore, the barrier penetrability D was calculated for each subinterval using the well-known formulae [10]

$$D = \begin{cases} \dfrac{\sinh^2(\pi ka)}{\sinh^2(\pi ka) + \cosh^2((\pi/2)\sqrt{32\pi^2 U_0 ma^2/h^2 - 1})} & \text{for } 32\pi^2 mU_0a^2/h^2 < 1 \\ \dfrac{\sinh^2(\pi ka)}{\sinh^2(\pi ka) + \cos^2((\pi/2)\sqrt{1 - 32\pi^2 U_0 ma^2/h^2})} & \text{for } 32\pi^2 mU_0a^2/h^2 > 1 \end{cases} \tag{5}$$

where $k = \sqrt{2mw}/\hbar$. We emphasize that equation (5) is the exact solution of the Schrödinger equation for the potential (3).

The product DN_w gives N_{w0}, or the number of particles from a given subinterval that overcome the barrier. The total sum of all of the N_{w0} gives N_0, which is the total number of particles of the ensemble transmitted through the barrier. Thus $F = N_0/N$ will be the effective barrier penetrability. Let us note that the magnitude of F is determined not only

† $U_{\max}$ was adjusted so as to lead to the neglect of a number of particles outside the interval for each given value of the temperature.

by the tunnelling but also by the over-barrier reflection. In fact, even in the case where the energy of the particle is larger than U_0, D may be less than 1.

The probability of thermal activation G can be easily found by calculating the fraction of particles with energy above U_0:

$$G = \int_{U_0}^{\infty} \frac{2}{\sqrt{\pi}} (k_B T)^{-1.5} \sqrt{w} \exp\left(-\frac{w}{k_B T}\right) \mathrm{d}w. \tag{6}$$

In numerical calculations, for the upper bound we of course make the substitution $\infty \rightarrow U_{max}$, with the provisos noted above.

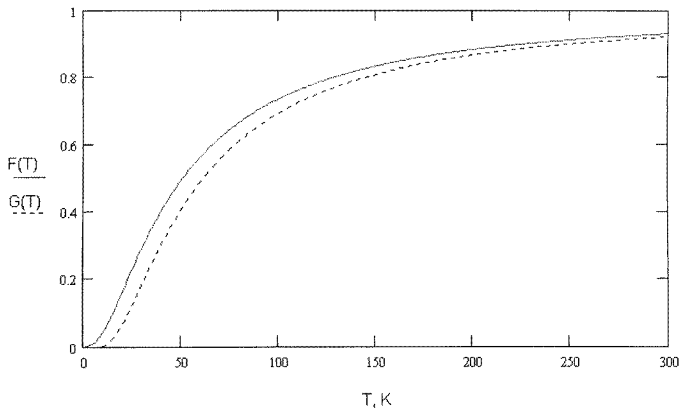

Figure 1. The effective quantum barrier penetrability $F(T)$ in comparison with the classical thermal activation probability $G(T)$ for the Bloch wall over a wide temperature region.

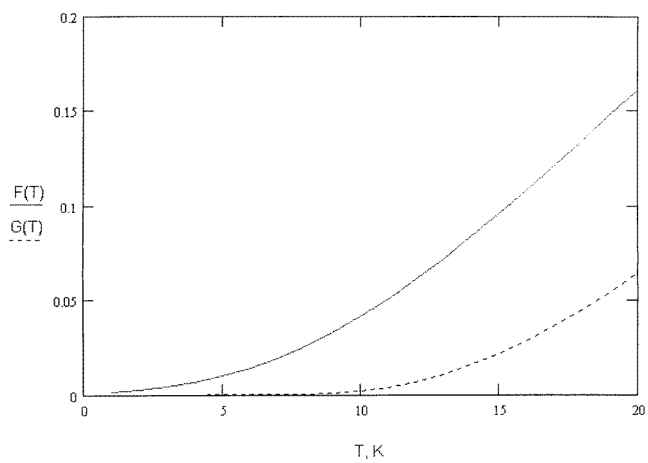

Figure 2. The effective barrier penetrability $F(T)$ and classical thermal activation probability $G(T)$ for the Bloch wall at low temperatures (up to 20 K).

The dependencies of both the effective barrier penetrability F and the probability of thermal activation G on the temperature are plotted in figures 1 and 2. Figure 1 shows the influence of the above-discussed mechanism over a wide temperature range from 0 to

300 K. In figure 2, for clarity, we have plotted just the results for the narrow region from 0 to 20 K, in which the greatest difference between F and G is found. We hope that these results demonstrate the severity of the exposure of the tunnelling to the depinning processes.

3. Thermostimulated domain wall tunnelling in a weak ferromagnets; accounting for the quasi-relativistic phenomena

As stated above, the thermal stimulation of tunnelling through Bloch walls can be realized only under rigid restrictions, especially as regards the parameter U_0. For high barriers, the condition $U_0 \ll mc^2$ may be broken and the structure of the wall will be varied; quasi-relativistic phenomena arise for this reason. How to account for such peculiarities will be demonstrated by means of an example: walls in a weak ferromagnet. A theory which successfully described the high-energy dynamics in such materials was evolved in reference [11]. Weak ferromagnets have very suitable properties for comparison of theory with experimental data. Walls in weak ferromagnets, as a rule, have masses one or two orders of magnitude smaller then those of walls in ferromagnets, and their widths are significantly smaller too. Both of these factors lead to increases of the tunnelling rate. Let us also note that very pure samples with low defect concentrations are available now; this leads to good reproducibility of the results of measurements.

3.1. The model

Let us consider, for example, a weak ferromagnet of terbium orthoferrite type within the two-lattice approximation using the ferromagnetic and antiferromagnetic vectors $\boldsymbol{m}$ and $\boldsymbol{l}$, respectively. Its thermodynamical potential will be [12]

$$\Phi_0(\boldsymbol{l},\boldsymbol{m}) = Jm^2 + A(\boldsymbol{\nabla}\cdot\boldsymbol{l})^2 - \boldsymbol{m}\cdot\boldsymbol{H} + d_1 m_x l_z - d_3 m_z l_x + K_{ac} l_z^2 + K_{ab} l_x^2$$

where J and A are, respectively, the uniform and non-uniform exchange constants, K_{ac} and K_{ab} are the anisotropy constants, $\boldsymbol{H}$ is the total external field acting on the wall, and d_1 and d_3 are the Dzyaloshinsky exchange constants. After minimization with respect to $\boldsymbol{m}$, one obtains [12, 13]

$$\Phi_0 = A(\boldsymbol{\nabla}\cdot\boldsymbol{l})^2 - \frac{\chi_\perp^2}{2}(H^2 - (\boldsymbol{H}\cdot\boldsymbol{L})^2) - M_z^0 H_z l_x - M_x^0 H_x l_z + K_{ac} l_z^2 - K_{ab} l_x^2$$

where $\chi_\perp$ is the transverse susceptibility, and M_x^0 and M_z^0 are the values of the magnetization in the phases $\Gamma_4(\boldsymbol{l} \parallel \boldsymbol{x})$ and $\Gamma_2(\boldsymbol{l} \parallel \boldsymbol{z})$ respectively. Without loss of generality, we can consider ac-type walls only. In spherical coordinates, the corresponding Lagrange density will be [15]

$$\mathcal{L} = \frac{\chi_\perp}{2\gamma^2}\left(\frac{\partial \boldsymbol{l}}{\partial t}\right)^2 - \frac{\chi_\perp}{\gamma}\boldsymbol{H}\left[\boldsymbol{l}, \frac{\partial \boldsymbol{l}}{\partial t}\right] - \Phi_0.$$

The appropriate Lagrange function per unit area of the domain wall has the essentially quasi-relativistic form [11]

$$L = -m^* c^2 \sqrt{1 - v^2/c^{*2}} - U(x) \qquad (7)$$

where $m^* c^{*2} = 4\sqrt{AK}$ and where $c^* = \gamma\sqrt{A/\chi_\perp}$ is the spin-wave velocity; $U(x)$ is the potential of the wall–defect interaction. The Hamiltonian associated with equation (7) will then be

$$H_g(p, x) = c^* \sqrt{p^2 + m^{*2} c^{*2}} + U(x) \qquad (8)$$

where

$$p = \frac{mv}{\sqrt{1 - v^2/c^{*2}}}$$

is the canonical impulse. The kinetic energy will then be $K = pc^*$.

The quasi-relativistic form of Hamiltonian (8) contains the essential peculiarities of the wall dynamics. It is important for us that the energy distribution for the walls (quasiparticles) will be appreciably changed, and that the effective barrier penetrability F and the probability of thermally activated depinning G may be changed too. Therefore it is necessary to define an energy distribution for quasi-relativistic particles. For this purpose we shall use the Gibbs canonical distribution

$$\mathrm{d}z_p = a^0 \exp(-K(p)/k_B T)\ \mathrm{d}p$$

where a^0 can found by using the normalization condition $\int_{-\infty}^{\infty} \mathrm{d}z_p = 1$. Since $K(p) = cp$,

$$z(w) = \frac{1}{2(k_B T)^3} w^2 \exp\left(-\frac{w}{k_B T}\right). \tag{9}$$

Obviously, the maximum of this distribution, in comparison with that for a classical distribution, will be shifted into a higher-energy region. Therefore the fraction of 'vigorous' particles will increase, and F and G will correspondingly change.

3.2. Results of the calculations

From the results of numerical calculations detailed below, it is apparent that the walls in weak ferromagnets are in fact very suitable subjects for the study of tunnelling in the high-temperature range. Let us give the main parameters. We will take the effective mass of a wall to be $m = 10^{-13}$ g cm^{-2}. We shall take the width of the energy barrier of the defect and the width of the wall Δ to be both of the order of 10^{-6} cm. The area of the defect and the area of the tunnelling segment of the wall will both be taken as $S = 10^{-13}$ cm^2; hence the quasiparticle mass will be $m^* = 10^{-26}$ g. The height of the barrier can be found from the value of the coercive force [6]: $U_0 = 10^{-13}$ erg. On the face of it, such values seem scarcely suitable for tunnelling: after formal substitution of these parameters in equation (5), one finds the negligibly small range for D of about 10^{-80} or less. However, taking into account the quasi-relativistic situation essentially changes the physics.

Let us consider this situation in more detail. Now, as before, we will carry out modelling of the barrier using the function (3). We will sort the particles of an ensemble by energy according to the algorithm presented in section 2. But taking into account equation (9), we find now for N_w

$$N_w = \int_{w-\delta w/2}^{w+\delta w/2} \frac{N w^2}{2(k_B T)^3} \exp\left(-\frac{w}{k_B T}\right)\ \mathrm{d}w. \tag{10}$$

Accordingly, the probability of thermally activated depinning will be

$$G = \int_{U_0}^{\infty} \frac{w^2}{2(k_B T)^3} \exp(-w/k_B T)\ \mathrm{d}w.$$

The calculations of D for a wall in a weak ferromagnet were carried out numerically using the technique suggested in reference [6]. The final computational results for F and G are plotted in figure 3. It is apparent that with the chosen parameters the difference between the probability of quantum thermally stimulated depinning F and thermally activated depinning G is larger than for normal ferromagnets. In particular, at 50 K, F is twice

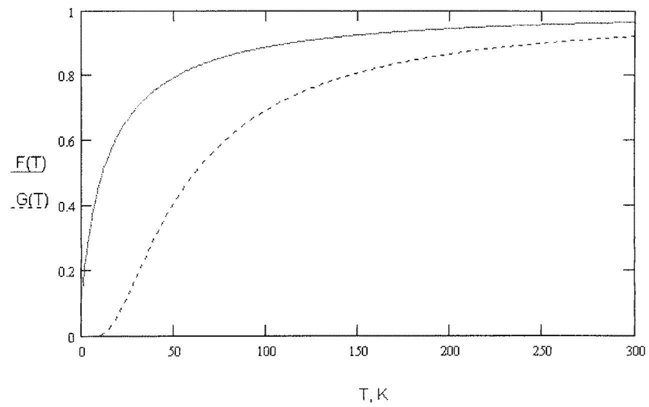

Figure 3. The effective quantum barrier penetrability $F(T)$ and the probability of thermally activated depinning $G(T)$ for a wall in a weak ferromagnet.

the size of G and, even at room temperature, the difference between F and G amounts to 0.05. Thus, taking account of quantum effects in the mechanism of the depinning is actually essential.

In section 2 we considered Bloch wall tunnelling in normal ferromagnets. The results obtained for that case are valid for low-energy walls only. For high-energy walls the dynamics also becomes quasi-relativistic and for its description the Walker [7] technique is necessary. The structure of the Walker solution corresponds formally to Hamiltonian (9); therefore one might think it reasonable to extend the results of section 3 to Bloch walls. However, the nature of the maximal velocity in this case is different. In ferromagnets, when the wall velocity tends to its maximal (not limiting) value, the form of the wall can be essentially changed. In this case it is necessary to take into account an additional energy, connected with leakage fields, because extension of the results of section 3 to Bloch walls requires additional analysis.

4. The feasibility of experimental tests

Domain wall depinning via tunnelling is usually investigated at very low temperatures (see, for example, the review [4]). Unfortunately, we have no data on tunnel depinning at high temperatures. The data concerning the basic parameters of a problem, such as the form, width and height of the energy barrier, were obtained indirectly and, at the present time, are not very reliable. This made it difficult to compare theory with experimental data. Therefore, special importance attaches to experiments in which the depinning of solitary walls is investigated. The problems connected with the statistical nature of depinning will in this case be eliminated. In this respect, the experimental technique used in reference [5] is very interesting: a solitary wall tunnelling in a superthin wire was investigated. In our view, extension of this technique to the high-temperature region may be very useful for testing the validity of the physical mechanism described in the present paper. Testing of the results presented could also be carried out using the magnetic noise technique. Clearly, there is an urgent need for a search for departures from the classical temperature dependence for any physical quantities associated with wall depinning.

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